

### Problem 3.10

Is the ground state of the infinite square well an eigenfunction of momentum? If so, what is its momentum? If not, *why* not? [For further discussion, see Problem 3.34.]

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#### Solution

The ground state of the infinite square well is

$$\psi_1(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right),$$

and the momentum operator is

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}.$$

Check to see if the ground state is an eigenfunction of the momentum operator.

$$\hat{p}\psi_1(x) = -i\hbar \frac{\partial}{\partial x} \left( \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a} \right) = -i\hbar \left( \frac{\pi}{a} \right) \left( \sqrt{\frac{2}{a}} \cos \frac{\pi x}{a} \right) \neq p\psi_1(x)$$

Since  $\hat{p}\psi_1(x)$  does not evaluate to a constant times  $\psi_1(x)$ , the ground state of the infinite square well is not an eigenfunction of the momentum operator.